# Applied Practice
# in

# *Accumulation and the Fundamental Theorem of Calculus*

## *AP\**

# RESOURCE  GUIDE

# APPLIED PRACTICE
## Resource Guide
### *Accumulation and the Fundemental Theorem of Calculus*
### AP* Version

## Teacher Overview and Guide

## Student Practices

## Answer Keys, Explanations and Scoring Guidelines

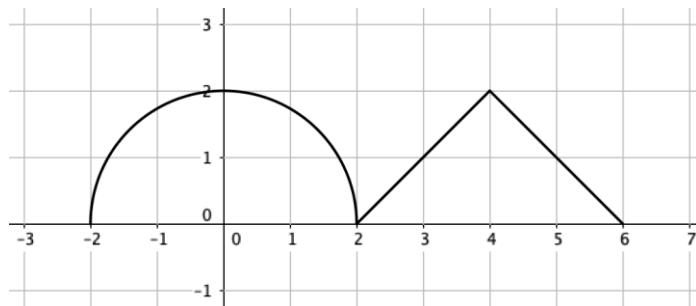*No Calculator Allowed*

**Level I:**

1. Evaluate $\int_0^2 x \cdot e^{x^2}\ dx$.

   A. $2e^4$

   B. $e^4 - 1$

   C. $\dfrac{e^4-1}{2}$

   D. $\dfrac{e^4}{2}$

2.



Graph of $f'$

The graph of $f'$ consists of a semicircle and two line segments as shown above.
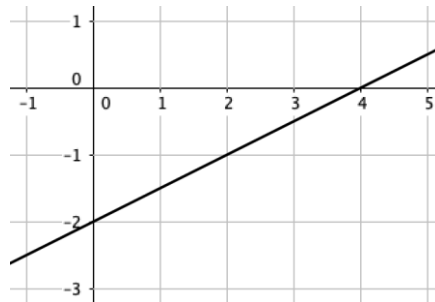
Given that $f(0) = 4$, find $f(6)$.

   A. $\pi + 4$

   B. $\pi + 6$

   C. $\pi + 8$

   D. $2\pi + 8$

3. $\int_{-2}^{x} 3t^2 + 2t\ dt =$

   A. $x^3 + x^2$

   B. $3x^2 + 2x$

   C. $x^3 + x^2 + 4$

   D. $3x^2 + 2x + 12$

4. Find the average value of $y = 2x - x^2$ on the interval $0 \le x \le 2$.

    A.  0

    B.  $\dfrac{1}{2}$

    C.  $\dfrac{2}{3}$

    D.  $\dfrac{4}{3}$

5.

Graph of $f'$

The graph of $f'$ consists of the line shown above.

Given that $f(0) = 5$, find $f(4)$.

    A.  –4

    B.  1

    C.  4

    D.  9

6. A particle moves along the $x$-axis such that its initial velocity $v(0) = 5\ ft/sec$ and acceleration is given by $a(t) = 2t + 1\ ft/sec^2$. Find the velocity of the particle at time $t = 4$ seconds.

    A.  $2\ ft/sec$

    B.  $9\ ft/sec$

    C.  $20\ ft/sec$

    D.  $25\ ft/sec$

**Level II:**

35. Let $G(x) = \int_0^{x^2} ln(t^3 + 1)\, dt$. Find $G'(2)$.

    A. $4.174$
    B. $8.232$
    C. $8.789$
    D. $16.698$

36. If $f'(x) = ln(x^2 - 2)$ and $f(5) = 8$ then $f(2) = $

    A. 0.693
    B. 1.379
    C. 6.621
    D. 14.621

37. Water is flowing into a tank at a rate of $F(t) = \frac{10}{t+1}$ $gallons/min$. If there were 20 gallons in the tank at time $t = 0$, How much water is in the tank after 10 minutes?

    A. 20.909 gallons
    B. 23.979 gallons
    C. 40.909 gallons
    D. 43.979 gallons

38. At midnight, there are 1.2 inches of snow on the ground. If the snow continues to fall at a rate of $R(t) = 1 - 0.5t\, cos(0.85t)$ inches per hour where $t$ is the number of hours since midnight, what is the depth of the snow at 5:00 AM?

    A. 2.115 inches
    B. 3.315 inches
    C. 8.633 inches
    D. 9.833 inches

**A graphing calculator is required on the following problem**

2.

The rate at which water is flowing into a tank is modeled by the function $R$, where $R(t) = 18sin\left(\frac{t^2}{40}\right)$ where $t$ is measured in hours and $R$ is measured in cubic feet per hour and $0 \le t \le 10$. There is a drain on the tank that allows water to escape at a rate of $D(t) = \frac{-12(x-7)^2}{49} + 12$ cubic feet per hour, for $0 \le t \le 10$. There are 50 cubic feet water in the tank at time $t = 0$.

A.  How many cubic feet of water enter the tank during the 10-hour time interval $0 \le t \le 10$?

B.  Is the amount of water in the tank increasing or decreasing at time $t = 6$ hours? Give a reason for your answer.

C.  For what time $t$, $0 \le t \le 10$, is the amount of water in the tank a minimum? Justify your answer.

D.  Write, but do not solve, an equation involving one or more integrals that gives the time $t > 0$ for which the amount of water in the tank is equal to 52 cubic feet.

Explanations for Multiple-Choice Questions:

| Q1 | |
|---|---|
| A | The student may have evaluated $xe^{x^2}$ at $x = 2$. |
| B | The student may have incorrectly applied u substitution. |
| **C** | **This answer is correct.** |
| D | The student may have mistakenly set $e^0 = 0$. |

| Q2 | |
|---|---|
| A | The student may have not added $f(0) + 4$. |
| B | The student may have calculated the area of the triangle incorrectly. |
| **C** | **This answer is correct.** |
| D | The student may have calculated the area of the quarter circle incorrectly. |

| Q3 | |
|---|---|
| A | The student may have forgot to evaluate the antiderivative at $x = -2$. |
| B | The student may have mistakenly taken the derivative of the correct answer. |
| **C** | **This answer is correct.** |
| D | The student may have substituted $x$ and $-2$. |

| Q4 | |
|---|---|
| A | The student may have computed the average rate of change or may not have integrated. |
| B | The student may have found the wrong antiderivative. |
| **C** | **This answer is correct.** |
| D | The student may have forgot to divide by 2. |

| Q5 | |
|---|---|
| A | The student may have found $\int_0^4 f'(x)dx$. |
| **B** | **This answer is correct.** |
| C | The student may have set the integral to 4 (not $-4$) and may not have added $f(0)$. |
| D | The student may have set the integral to 4 (not $-4$). |

| Q6 | |
|---|---|
| A | The student may have taken the derivative of the acceleration. |
| B | The student found $a(4)$. |
| C | The student did not use $v(0) = 5$. |
| **D** | **This answer is correct.** |

| Q35 | |
|---|---|
| A | The student may have not applied the chain rule. |
| B | The student may have computed G(2). |
| C | The student may have not substitute $x^2$ but did apply the chain rule. |
| **D** | **This answer is correct.** |

| Q36 | |
|---|---|
| A | The student may have evaluated $f'(2)$. |
| **B** | **This answer is correct.** |
| C | The student may have evaluated $\int_2^5 \ln(x^2 - 2)\, dx$. |
| D | The student may have added $8 + \int_2^5 \ln(x^2 - 2)\, dx$. |

| Q37 | |
|---|---|
| A | The student evaluated $20 + F(10)$ |
| B | The student did not account for the 20 gallons initially in the tank |
| C | The student evaluated $20 + 20 + F(10)$ |
| **D** | **This answer is correct.** |

| Q38 | |
|---|---|
| A | The student found the rate at which the snow was falling at 5:00 A.M. |
| B | The student found the rate at which the snow was falling at 5:00 A.M. plus 1.2. |
| C | The student did not add the initial 1.2 inches. |
| **D** | **This answer is correct.** |

Free-Response #2 Scoring Rubric

| | |
|---|---|
| A. $\int_0^{10} R(t)dt = 94.994$ | 1: integrand <br> 1: answer |
| B. $R(6) - D(6) = 2.34478 > 0$ <br> Since $R(6) > D(6)$, the amount of water in the tank is increasing at time $t = 6$ hours. | 1: considers $R(6)$ and $D(6)$ <br> 1: answer and reason |
| C. <br> The amount of water in the tank at time $t$, $0 \le t \le 10$, is given by <br> $50 + \int_0^t (R(x) - D(x))dx$ <br> $R(t) - D(t) = 0 \Rightarrow t = 0, 5.1798652$ <br><br> <table><tr><td>$t$</td><td>Amount of water in tank</td></tr><tr><td>0</td><td>50</td></tr><tr><td>5.1798652</td><td>35.356</td></tr><tr><td>10</td><td>55.198</td></tr></table> <br><br> The amount of water in the tank is a minimum at $t = 5.179$ (or 5.180) hours. | 1: considers $R(t) - D(t) = 0$ <br> 1: answer <br> 1: justification |
| D. $50 + \int_0^t (R(t) - D(t))dt = 52$ | 1: integral <br> 1: equation |