Applied Practice
in

Particle Motion
*AP**

RESOURCE GUIDE

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APPLIED PRACTICE
Resource Guide
Particle Motion
AP* Version

Teacher Notes and Strategies

Student Practices and Assessment

Answer Keys, Explanations, and Rubrics

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The following are the concepts and formulas that will be needed for Particle Motion on the AP Calculus Exam.

All time intervals below are $a \le t \le b$

$$Total\ Distance = \int_a^b |v(t)|dt$$

$$Final\ Position:\ x(b) = x(a) + \int_a^b v(t)dt$$

$$Speed = |v(t)|$$

$$Speed\ Increasing\ or\ Decreasing\ at\ t = c:\ \ Be\ sure\ to\ check\ BOTH\ v(c)\ and\ a(c)!$$

$$If\ v(c)\ and\ a(c)\ have\ SAME\ signs \rightarrow speed\ increasing$$

$$If\ v(c)\ and\ a(c)\ have\ OPPOSITE\ signs \rightarrow speed\ decreasing$$

$$Particle\ moving\ right\ or\ up:\ v(t) > 0$$

$$Particle\ moving\ left\ or\ down:\ v(t) < 0$$

$$Particle\ changes\ direction:\ Find\ when\ v(t) = 0\ and\ check\ for\ sign\ change\ in\ v(t)$$

$$Average\ Value\ on\ [a,b]:\ \frac{1}{b-a}\int_a^b f(t)dt$$

$$Average\ Rate\ of\ change\ on\ [a,b]:\ \frac{f(b)-f(a)}{b-a}$$

Important Notes:

1. If asked to find the average of the equation that is GIVEN, use the Average Value formula.
2. If asked to find the average of the RATE of the given equation, use Avg. Rate of Change

Example: $If\ given\ v(t), to\ find$:

1. $Average\ velocity:\ Use\ average\ value\ theorem$
2. $Average\ acceleration:\ Use\ average\ rate\ of\ chagne$

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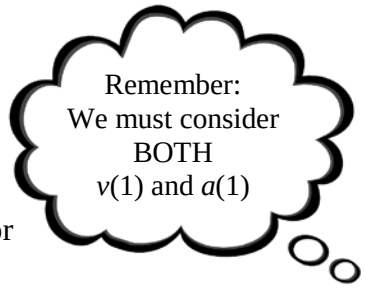
Free-Response #2 (**Calculator Required**)

For $t \geq 0$, a particle moves along the *x*-axis. The velocity of the particle at time t is given by $v(t) = 1.35 - e^{\cos(t)}$. The particle is at position $x = -3$ at time $t = 2$.

(a) At time $t = 1$, is the particle speeding up or slowing down? Justify your answer.

$v(1) =$ _____ $a(1) =$ _____



AP Exam Tip: On a Calculator active problem, be sure to use your technology to find $v(1)$ and $a(1)$. Do not compute values by hand or take derivatives/integrals by hand!

Use your values above to choose the appropriate answers to complete the blanks below.

Answer: At time $t = 1$, the particle is _____ because $v(1)$ and (1) have
Speeding up/ Slowing down

the _____ sign.
Same/ Opposite

(b) Find all times t in the interval $0 < t < 6$ when the particle changes direction. Justify your answer.

To find the times when the particle changes direction, we must meet two requirements. Choose the correct options from the boxes below that will lead to the correct answer:

1st Requirement	**2nd Requirement**
$x(t) = 0$	$x(t)$ changes sign
$v(t) = 0$	$v(t)$ changes sign
$a(t) = 0$	$a(t)$ changes sign

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No Calculators Allowed

Level I:

1. The position of a particle moving along the *x*-axis at any time t is given by $x(t) = 2t^2 - 4t + 1$. What is the acceleration of the particle when $t = 3$?

   (A) 0 (B) 4 (C) 7 (D) 8

2. A particle moves along a straight line such that its velocity at time t, $(t > 0)$, is given by $v(t) = t^2 - 6t - 16$. Find the velocity of the particle when its acceleration is 0.

   (A) -25 (B) -16 (C) 3 (D) 8

Use the table below to answer questions 3-5.

t	0	1	4	6	10
$v(t)$	0	-2	1	-3	5

The table above gives selected values of a bug's velocity, in ft/min, that is moving in a straight path over the interval $0 \leq t \leq 10$ minutes.

3. Find the average acceleration of the bug, in ft/min^2, over the ten minutes.

(A) -0.25 (B) 0.1 (C) 0.5 (D) 1

4. Approximate the acceleration of the bug, in ft/min^2, at time $\text{t} = 3$ min.

(A) -0.5 (B) 0.25 (C) 0.5 (D) 1

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Use the table below to answer questions 36-37

t	0	1	3	5
$v(t)$	-2	1	4	0

A bug is walking along a straight path for $0 \leq t \leq 5$. The velocity of the bug at four select times is given in the table above. At time $t = 5$, the bug is at position -2.

36. Using a trapezoidal sum with three subintervals, approximate the average velocity of the bug over the interval [0, 5].

(A) 8.5 (B) 1.7 (C) 0.6 (D) 0.4

37. Use a right Riemann sum with three subintervals to approximate the position of the bug at time $t = 0$.

(A) -8 (B) 4 (C) 6 (D) -12

38. A bug is walking along a straight path for $0 \leq t \leq 3$ with position $s(t) = sin\left(\frac{\pi t}{2}\right)$. During which interval(s) is the bug speeding up?

(A) (0, 1) (B) (1, 2) (C) (2, 3) (D) (0, 1) U (2, 3)

The following problems require a graphing calculator

Level I:

39. The velocity of a particle moving along the *x*-axis is given by $v(t) = 2.3t\ cos(t^2)$. Find the acceleration of the particle at the time $t = 1$.

(A) -2.628 (B) -0.696 (C) 0.968 (D) 1.243

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